MODELING RAPIDLY ROTATING STARS

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Abstract. We review the quest of modeling rapidly rotating stars during the past 40 years and detail the challenges to be taken up by models facing new data from interferometry, seismology, spectroscopy... We then present the progress of the ESTER project aimed at giving a physically self-consistent model for the structure and evolution of rapidly rotating stars.

1 Introduction

The recent observations of rapidly rotating stars with optical and infra-red interferometers has renewed the interest in modeling these kind of stars (Domiciano de Souza et al. 2002; Roxburgh 2004; Jackson et al. 2005). Indeed, these data now reveal not only the shape of these stars (Domiciano de Souza et al. 2003) but also the emissivity distribution of the atmosphere (Domiciano de Souza et al. 2005; Peterson et al. 2006a; Peterson et al. 2006b). With such an information, models can be adjusted and the inclination of the rotation axis can be deduced as well as the true equatorial velocity of the star. Realizing that the brightness of a star depends on the orientation of its angular momentum vector (equatorial regions are cooler than polar ones), one wonders what is the real position of rotating stars in the Herzsprung-Russell diagram, what has been their evolution and therefore what is their age. The recent discovery (Peterson et al. 2006b) that Vega, a photometric standard, is a rapid rotator seen pole on makes the foregoing questions all the more exciting.

In this review, I would like to briefly present a short history of models of rotating stars and then describe the challenges to be taken up. When I write rotating stars I assume that rotation is not negligible: I therefore drop the term 'rapidly' since the classification of a star in the category of rapid rotators is essentially a matter of precision (for instance, very precise measurements of frequency of oscillations require the use of non-perturbative models at much slower rotations than less resolved data, see Reese et al. 2006). All approaches based on asymptotically slow rotations will not be considered. I will then give a brief description of the physical phenomena that should be taken into account when modeling rotating stars; the ESTER project which aims at forecasting the evolution of rotating stars is then presented. A brief discussion concludes this short review.

2 A brief history of models

The quest of models describing rotating stars up to the break-up limit (i.e. when the equatorial velocity reaches the keplerian one), really started with the work of James (1964) who calculated the structure of self-gravitating polytropes at any rotation rate in the permitted range.

This pioneering work was rapidly followed by an attempt from Roxburgh et al. (1965) who went beyond polytropic models using a partition of the star in two regions: the internal one where centrifugal effect was assumed small and the external where the Roche approximation could be used (no self-gravity). Some simple microphysics could be taken into account (nuclear heating and Kramers type opacities). But with such an approach the rotation rate was limited and the radiative zone assumed in a hydrostatic state (which is not possible physically, see below).

The next step was undertaken by an American team (P. Bodenheimer, J. Ostrikers and collaborators) who published a series of 8 papers which introduced and exploited the Self-Consistent Field method (Ostriker &

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Mark 1968; Ostriker & Bodenheimer 1968; Mark 1968; Ostriker & Hartwick 1968; Jackson 1970; Bodenheimer & Ostriker 1970; Bodenheimer 1971; Bodenheimer & Ostriker 1973). The idea of this method is to use the solution of Poisson's equation (for the gravitational potential ϕ) expressed with the Green function, i.e.

$$\phi(\vec{x}) = -G \int \frac{\rho(\vec{x}')}{|\vec{x} - \vec{x}'|} d^3\vec{x}'$$

which has the great advantage of including the boundary conditions on ϕ at infinity. Again, all dynamics was sacrificed to investigate the gross role of rotation on bulk quantities of a star (e.g. luminosity, life time, ...). But the authors faced many difficulties, the code being not flexible enough to deal with masses less than 9 $M_\odot$ or with very rapid rotation. Precision of the calculations was evaluated by the virial test (see sect. 4) and was found to be $4\,10^{-3}$ (Jackson 1970).

Soon after, M. Clement took up the challenge (Clement 1974). His method was based on a resolution of the 2D Poisson equation with finite differences. Compared with preceding ones, results were better at small masses and problems appeared at large masses. Similarly, the star was forced to a hydrostatic barotropic state. There too, the virial gave an estimate of the precision which was found around $2\,10^{-4}$. Clement (1978) examined the effects of prescribed differential rotation and, Clement (1979) boundary conditions with a grey atmosphere (among others). Lastly, Clement (1994) investigated the case of differentially rotating massive stars.

Meanwhile, the Japanese school lead by Y. Eriguchi started a new attempt with the aim of computing relativistic configurations. An original method (called EFGH) was first devised (see Eriguchi 1978) but was rapidly abandoned because of its difficulty to cope with discontinuities. A new method (Eriguchi & Sugimoto 1981), close to the Self-Consistent field one, has then been used to go on the route towards more realistic models. The first consideration of a baroclinic star appeared with the work of Uryu & Eriguchi (1994, 1995); this first attempt introduced some baroclinic flow in the radiative zone but the neglect of viscosity left some degeneracy in the model (see Rieutord 2006). Ultimately, Shindo et al. (1997) focused on improving microphysics but relaxed on the dynamics keeping a barotropic configuration.

Parallel to this work Eriguchi & Müller (1985, 1991) developed a code based on a mapping of the star where the new radial variable ζ introduces surface coordinates similar to the surface of the star

$$r_i(\theta_k) = \zeta_i R_s(\theta_k)$$

with ζ_i regularly spaced between 0 and 1. Discretization is with finite differences radially and angularly. The code seems to be very robust being able to compute configurations very far from sphericity but the precision of calculations again tested with the virial theorem seems to be around $5\,10^{-4}$. In Eriguchi & Müller (1991), a meridional flow was computed; however, as the momentum equation was not solved for this flow, this flow should be considered as a mere illustration of the thermal disequilibrium brought about by the imposed barotropicity.

As mentioned in the introduction, most recent work has been published by Roxburgh (2004) and Jackson et al. (2004, 2005). Models are still barotropic but microphysics is improved as well as the precision of the models. Jackson et al. (2005) report a virial test at 10^{-5}.

The conclusion of 40 years of research on modeling rotating stars is that still large parts of the problem remain unexplored. None of the models have reached the state of computing self-consistently a baroclinic radiative zone, which is unavoidable in a rapidly rotating star (like Vega or Altair). Nobody has ever considered the case of evolution under fast rotation except with spherically symmetric model (Maeder & Meynet 2000). These are very difficult questions but the recent observational result of interferometry as well as the forthcoming data from seismology (COROT satellite) force us to go beyond these obstacles.

3   The ideal model for rotating stars

The foregoing review of the attempts made at modeling rotating stars showed us that models are still quite far from actual stars. The reader may wonder how far. This question brings us to the description of the ideal model for a rotating star.

Such a model should describe the mean state of a star at any time of its life and especially the new quantity specific to these stars: angular momentum.

Unlike a non-rotating star which is a one-dimensional object (in a large-scale description) which needs only scalar fields (I forget magnetic fields), a rotating star is, at least, a two-dimensional object with, at least, one

vector field in addition to all scalar field. Hence, complexity increases not only by the multi-dimensional nature of the model but also by the number of physical quantities to be determined. This implies that the ideal model deals consistently with angular momentum and especially the losses and gains through stellar winds and accretion. Such a model should also take into account the baroclinicity of radiative zones and there, the anisotropic turbulence which appears through shear instabilities; it should also include a mean-field theory of convection to forecast Reynolds stresses and heat flux. Of course, observers would like to know the emissivity of the atmosphere as a function of latitude (if they use interferometry) or as a function of wavelength if they do spectroscopy. But if they do asteroseismology they surely wish to know the eigenspectrum of these objects.

The foregoing points show that progress in the understanding of rotating stars needs also some advances in the following questions of stellar physics:

- How angular momentum is distributed in a star and how is it input or output with what consequences ?

- The immediately following question concerns the nature of the Reynolds stresses in the convective and radiative zones.

- Then, what is the baroclinic state of the radiative regions ?

- Similarly, the atmosphere is in a baroclinic state and cannot be at rest: how strong are the differential rotation and the meridional currents? Does the atmosphere develop strong azimuthal winds streaming around the star like Jupiter's winds?

- Gravity darkening can be so efficient that equatorial regions are cool enough to develop convection; this raises the question of the latitude dependence of emissivity of the atmosphere beyond the Von Zeipel model (see the attempt of Lovekin et al. 2006).

- I did not mention magnetic fields. Clearly they multiply the number of problems and first steps should ignore them if possible.

4   The ESTER project

Crazily enough, the ESTER project (Evolution STEllaire en Rotation) takes up the challenge of at least producing a physically self-consistent model of a rotating star at any rotation rate and let it evolve.

Technically, we construct a two-dimensional model using coordinates adapted to the geometry of the star (in fact to all its interfaces). Thus doing, we follow the work of Bonazzola et al. (1998) which uses a mapping between spheroidal coordinates and spherical coordinates (e.g. Rieutord et al. 2005; Reese et al. 2006). Such a mapping is necessary to impose correctly boundary or interface conditions.

Then we discretize the equations using spectral methods both radially, with Chebyshev polynomials, and horizontally, with spherical harmonics.

As our predecessors, we first controlled our model with polytropes in solid body rotation. The results have been compared successfully to those of James (1964) and then tested by the virial theorem which demands that

$$2T/W + 3P/W + 1 = 0 \qquad (4.1)$$

where

$$T = \int_{(V)} \frac{1}{2}\rho\Omega_*^2 r^2 \sin^2\theta dV, \qquad W = \frac{1}{2}\int_{(V)} \rho\phi dV, \qquad P = \int_{(V)} PdV$$

are respectively the kinetic, gravitational and internal energy. Typically, the virial equation is satisfied with a precision better than 10^{-8}. These very precise models turned out to be excellent for asteroseismological purposes (see Lignières et al. 2006; Reese et al. 2006).

In the next step we generalized the simple polytropes to stars composed of many polytropic layers (e.g. Rieutord et al. 2005).

However, polytropes are barotropic stars and they cannot describe the dynamics of radiative zones properly. Real rotating stars have indeed radiative zones which are the seat of slow baroclinic flows which show up as a differential rotation, a meridional circulation and certainly some weak anisotropic turbulence. As such flows were largely unknowns, we decided to first investigate them, *per se*, in some simplified context. Thus, in Rieutord
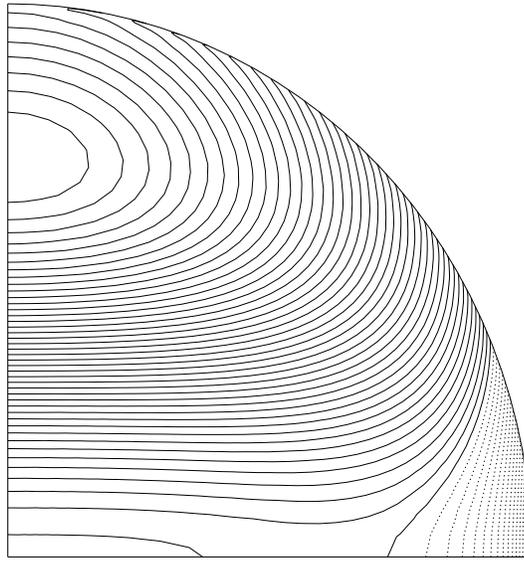
Fig. 1. The Brunt-Väisälä frequency squared in the meridional plane of a rapidly rotating star. Note the convectively unstable region near the equator ($N^2 < 0$).

(2006) we solved this problem in the case of spherically symmetric star made of a Boussinesq fluid, *i.e.* with a fluid of negligible compressibility. We could thus determine how the differential rotation of a radiative zone is controlled by the baroclinic torque and what was the role of the Ekman boundary layers. We also showed the dynamical role of the viscosity jump at the core-envelope boundary; it gives birth to a Stewartson layer parallel to the rotation axis which reaches surface layers. It was shown to be destroyed by molecular weight gradients which develop as the star evolves.

The next step has been to overcome the Boussinesq approximation and consider a more realistic radiative zone. For this purpose we enclosed a self-gravitating, compressible, viscous, rotating fluid inside a rigid sphere. Taking into account nuclear energy heating and radiative opacities in the Kramers form, we solved

$$\begin{cases} \Delta\phi = 4\pi G\rho \\ \rho T\vec{v}\cdot\nabla s = -\operatorname{div}\vec{F} + \varepsilon \\ \rho\left(2\vec{\Omega}\times\vec{v} + \vec{v}\cdot\nabla\vec{v}\right) = -\nabla p - \rho\nabla\left(\phi - \frac{1}{2}\Omega^2 r^2 \sin^2\theta\right) + \mu(\Delta\vec{v} + \frac{1}{3}\vec{\nabla}\operatorname{div}\vec{v}) \\ \operatorname{div}(\rho\vec{v}) = 0 \end{cases} \tag{4.2}$$

where we used the microphysics

$$\varepsilon = \varepsilon_0 X^2 \rho^2 T^{-2/3} \mathrm{e}^{-bT^{-1/3}}, \qquad \vec{F} = -\frac{16\sigma T^3}{3\kappa\rho}\vec{\nabla}T$$

In order to describe the radiative transport of energy, we use the opacity given by a Kramer's law $\kappa = \kappa_0 T^{-\beta}\rho^\eta$

A detailed description of the results may be found in Espinosa & Rieutord (2006). We show in Fig. 1 a meridian view of the distribution of the squared Brunt-Väisälä frequency . The remarkable results is the appearance, in the equatorial region, of convectively unstable fluid ($N^2 < 0$). A region which disappears if the rotation is slow enough.

5   Discussion

The results of Espinosa & Rieutord (2006) show that it is now possible to compute in a self-consistent manner a rotating radiative zone in a steady state, with realistic opacities, nuclear reactions and velocity fields. Yet,

the star is confined in a spherical box which supports some pressure. The next step is of course to relax this constrain and move to spheroidal geometry. Then it will be possible to apply more realistic boundary conditions on the temperature or plug a model of atmosphere. However, before that, we still need to find a good (or not too bad) model for the convection zones. Such a model should generalize in two dimensions, the approach of the mixing length theory and give a reasonable account of the convective flux.

Then, after some calibration tests using one-dimensional models, more physics will be taken into account. Most difficult steps will concern all transport phenomena which are related to turbulence. Indeed, turbulence controls the Reynolds stresses and thereby the diffusion of angular momentum which is closely related to the diffusion of elements (Zahn 1992).

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