

## IMPROVED ANGULAR MOMENTUM EVOLUTION MODELS FOR SOLAR-LIKE STARS

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**Abstract.** We present improved models for the angular momentum evolution of solar-like stars between 1 Myrs and 10 Gyrs. The models include a new braking law based on numerical simulations of magnetized stellar winds from Matt et al. (2012). Specific dynamo and mass-loss prescriptions are adopted to tight them to angular velocity. We assume constant angular velocity during the disk accretion lifetime and the models allow for core-envelope decoupling with a coupling timescale that varies between slow and fast rotators (Bouvier 2008). Stellar structure models of solar-mass stars are taken from Baraffe et al. (1998). We developed rotational evolution models for slow, median and fast rotators from the pre-main sequence (PMS) to the age of the Sun. The models are compared to the distributions of rotational periods observed for solar-type stars between 1 Myr and 5 Gyr in a number of star forming regions and young open clusters. We find that the model parameters accounting for the slow and median rotators are very similar to each other but differ significantly for fast rotators. We speculate that these differences may be related to different magnetic topologies in slow/median and fast rotators.

Keywords: Stars: solar-type, evolution, rotation, mass-loss, magnetic field

### 1 Introduction

Studying the angular velocity evolution is fundamental to understand how the stars lose and gain angular momentum during their life. For example, at its formation the angular momentum of the Sun was about thousand times greater than the current one which means that a large fraction of the initial angular momentum is lost by the stars between 1 Myr and the age of the Sun. The aim here is to understand how this huge quantity of angular momentum is extracted from the star, i.e., to highlight the involved physical processes. Stars lose angular momentum through two main ways 1) thanks to interactions with their surrounding disk which is expected to extract a large amount of angular momentum from them and 2) thanks to stellar winds which produce a braking torque on the stellar surface. Many theoretical advances about both the star/disk interaction (Ferreira et al. 2000; Matt & Pudritz 2005a,b; Zanni & Ferreira 2009; Matt et al. 2010; Zanni & Ferreira 2011) and the stellar wind (Matt & Pudritz 2008a,b; Matt et al. 2011) have been made since the first works of Ghosh & Lamb (1978) and Kawaler (1988), which provide us with a better comprehension of the angular momentum gain/loss processes. Stars evolve through three main stages namely the pre-main sequence (PMS), the zero age main-sequence (ZAMS) and the main-sequence (MS). On the PMS, during the first 10 Myr of their life, the stars are believe to be magnetically linked to their disk that results in a transfer of angular momentum  $J \approx \Omega_* I$  (where  $I$  is the moment of inertia and  $\Omega_*$  the angular velocity of the star) between the two. In response to this interaction the angular velocity of the convective envelope seems to remain more or less constant during all the disk lifetime (Bouvier 2008; Irwin & Bouvier 2009). Nowadays, this star/disk interaction still not has an accepted explanation but it is clear that it is needed in order to explain the small rotation periods observed for CTTs which are on average below  $\approx 8$  days, corresponding to angular velocity  $\leq 10$  % of their break-up speed. After the disk dissipation ( $\approx 1$ -10 Myr), the stars are free to spin-up, because of the stellar contraction, until they reach the ZAMS at about 30 Myr where the contraction stops. Then, the stars reach the MS where the braking by stellar wind becomes dominant so that they will slowly spin-down. In the model we used the Matt et al. (2012) breaking law which gives the angular momentum loss rate as a function of the stellar parameters ( $B_*$ ,  $\Omega_*$ ,  $R_*$ ,  $\dot{M}_{wind}$ ). We also used the magnetic dynamo relationship and the mass loss rate prescription of Cranmer & Saar (2011).

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## 2 Model assumption

### 2.1 Internal structure

Low mass stars are composed of two regions : an inner radiative core and an outer convective envelope. We follow MacGregor & Brenner (1991) by assuming that both the core and the envelope rotate as solid bodies but with different angular velocity. The amount of angular momentum  $\Delta J$  to be transferred from the core to the envelope in order to balance their angular velocities is given by  $\Delta J = (I_{env}J_{core} - I_{core}J_{env})/(I_{core} + I_{env})$  where  $I$  and  $J$  refer to the moment of inertia and angular momentum, respectively, of the radiative core and the convective envelope. As in Allain (1998), we assume that  $\Delta J$  is transferred over a time-scale  $\tau_{c-e}$ , which we refer to as the core-envelope coupling timescale.

### 2.2 Stellar winds

The angular momentum loss rate due to stellar winds can be expressed as  $dJ/dt \approx \Omega_* \cdot \dot{M}_{wind} \cdot r_A^2$  where  $r_A$  is the averaged value of the Alfvén radius that accounts for the magnetic lever arm,  $\Omega_*$  is the angular velocity at the stellar surface and  $\dot{M}_{wind}$  is the mass outflow rate. Most angular momentum evolution models so far have used Kawaler (1988) prescription to estimate the amount of angular momentum losses due to stellar winds, with some modifications such as magnetic saturation (Krishnamurthi et al. 1997; Bouvier et al. 1997) or a different dynamo prescription (Reiners & Mohanty 2012). The main difference between previous models and the ones we present here is that we base our estimates of angular momentum loss on the recent stellar wind simulations performed by Matt et al. (2012) who derived the following expression for  $r_A$ :

$$r_A = K_1 \left[ \frac{B_p^2 R_*^2}{\dot{M}_{wind} \sqrt{K_2^2 v_{esc}^2 + \Omega_*^2 R_*^2}} \right]^m R_* \quad (2.1)$$

where  $K_1 = 1.30$ ,  $K_2 = 0.0506$  and  $m = 0.2177$  are obtained from numerical simulations of a stellar wind flowing along the opened field lines of a dipolar magnetosphere.

We know that the magnetic strength does not follow a single power law with the velocity (Mestel & Spruit 1987; Saar 1996, 2001; Reiners et al. 2009) but instead it seems to saturated at a certain angular velocity threshold, which means that the magnetic activity becomes constant regardless of the angular speed of the star. We assume the stellar magnetic field to be dynamo generated, i.e., that the mean surface magnetic field strength scales to some power of the angular velocity. We thus have:  $f_* B_* \propto \Omega_*^b$  where  $b$  is the dynamo exponent (cf. Cranmer & Saar 2011),  $B_*$  is the strength of the magnetic field and  $f_*$  is the filling factor, i.e., the fraction of the stellar surface that is magnetized (cf. Reiners & Mohanty 2012). Magnetic field measurements suggests that the magnetic field strength  $B_*$  is proportional to the equipartition magnetic field strength  $B_{eq}$  :  $B_* \approx 1.13 B_{eq}$  where  $B_{eq}$  is define as

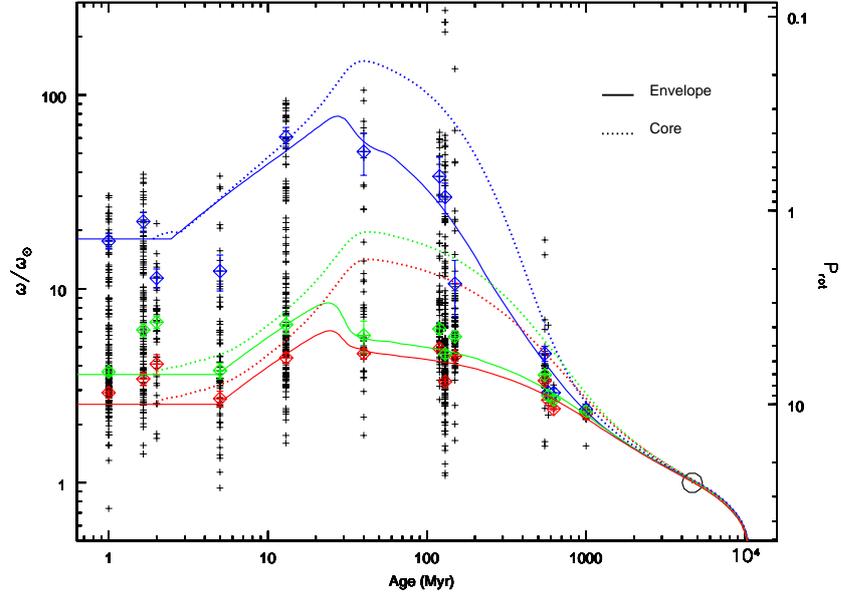
$$B_{eq} = \sqrt{\frac{8\pi\rho_* k_B T_{eff}}{\mu m_H}} \quad (2.2)$$

with  $\rho_*$  the photospheric density,  $k_B$  the Boltzmann's constant,  $T_{eff}$  the effective temperature,  $\mu$  the mean atomic weight and  $m_H$  the mass of a hydrogen atom. While the magnetic field strength appears to be more or less constant regardless of the angular velocity (Saar 1996; Cranmer & Saar 2011), the magnetic filling factor  $f_*$ , in contrast, strongly depends on the Rossby number  $Ro = P_{rot}/\tau_{conv}$  where  $\tau_{conv}$  is the convective turnover time. According to Saar (1996)  $f_* \propto P_{rot}^{-1.8}$  while Cranmer & Saar (2011) provides two different fits for  $f_*$  that are respectively the lower and upper shape of the  $f_*$ - $Ro$  plot :  $f_{min}$  that is the magnetic filling factor linked to the open flux tubes in non active magnetic region and  $f_{max}$  that is linked to the closed flux tubes in active regions. Their empirical fits give  $f_{min} \propto Ro^{-3.4}$  and  $f_{max} \propto Ro^{-2.5}$ . In the framework of our model the most relevant filling factor is obviously  $f_{min}$  that is related to the open flux tube which can carry matter through stellar outflow, thus  $f_* B_* \propto Ro^{-3.4} \propto \Omega_*^{3.4}$  and  $b \approx 3.4$ . The mean magnetic field strength saturation strongly depends on  $f_*$  and in the case where  $f_* = f_{min}$  the saturation occurs for  $\Omega_* \geq 10 \Omega_\odot$ . Observation of solar-type stars magnetic field suggests that saturation is reached when  $Ro < 0.1 - 0.13$  (see Reiners et al. 2009, Figure 6). For solar-like stars (i.e. spectral type G0-K5), Reiners et al. (2009) adopted  $\tau_{conv} \approx 20$  d, which is slightly larger than the empirical value of  $\tau_{conv} \approx 15$  d derived by Wright et al. (2011). With these estimates of  $\tau_{conv}$ , dynamo saturation is found to occurs at  $\Omega_{sat} \sim 10 - 16 \Omega_\odot$  which is coherent with the value of the saturation threshold found by Cranmer & Saar (2011). In Equation 2.1,  $B_p$  is the strength of the dipole magnetic field at

the stellar equator. Even though, the real stellar magnetic field is certainly not a perfect dipole we identify  $B_p$  to the strength of the mean magnetic field  $B_* f_*$ .

The mass-loss rates of solar-type stars at various stages of evolution are unfortunately difficult to estimate directly from observation. We therefore have to rely mainly on the results of numerical simulations of stellar winds, calibrated onto a few, mostly indirect, mass-loss measurements (e.g. Wood et al. (2002, 2005)). We used here the results from the numerical simulations of Cranmer & Saar (2011). Assuming that the wind is driven by gas pressure in a hot corona, as is likely the case for G-K stars, they found that  $\dot{M}_{wind} \propto \Omega_*^{2.4}$ . In our model we used directly the output of the subroutine BOREAS<sup>i</sup>, developed by Cranmer & Saar (2011), to get the mass loss rate  $\dot{M}_{wind}$  as a function of several stellar parameters like the angular velocity, the luminosity and the radius. Mass loss rate also appears to saturate at a certain velocity threshold (Holzwarth & Jardine 2007) and just like the magnetic field, the mass loss rate strongly depends on  $f_*$ . With  $f_* = f_{min}$ , the saturation of  $\dot{M}_{wind}$  appears at about  $10 \Omega_\odot$  corresponding exactly to the saturation threshold of the magnetic field. By combining Equations 2.1, the dynamo and the mass loss rate prescription with the expression of  $dJ/dt$ , we finally express the angular momentum loss-rate as a function of angular velocity.

### 3 Results



**Fig. 1.** Angular velocity evolution of the core and the envelope in the case of fast (blue), median (green) and slow (red) rotator models. The blue, red and green tilted square represents respectively the 90<sup>th</sup> quartile, the 25<sup>th</sup> quartile and the median of each rotation distributions. The open circle is the angular velocity of the present Sun.

Table 1 contains the parameters used for each rotator model presented in Figure 1. In this figure we plotted the angular velocity evolution for three rotator models and we confront them to the angular velocity distribution of 13 star forming regions and open clusters covering the age range from 1 Myr to 625 Myr plus the Sun (represented as an open circle in the figure).

### 4 Discussion

We present angular momentum evolution models of solar-like stars for slow, median and fast rotators from the pre-main sequence to the age of the present Sun and compare them to the distribution of rotational periods observed for solar-type stars between 1 Myr and 5 Gyr. By using the most recent physical advances about

<sup>i</sup><https://www.cfa.harvard.edu/~scanmer/Data/Mdot2011/>

**Table 1.** Model parameters

Parameter	Fast	Median	Slow
$P_{init}$ (days)	1.4	7	10
$\tau_{ce}$ (Myr)	12	28	30
$\tau_{disk}$ (Myr)	2.5	5	5
$K_1$	1.55	1.7	1.7
$m$	0.20	0.20	0.20

the stellar mass loss rate, wind braking and magnetic dynamo process we have developed rotational evolution models that reproduce reasonably well the observations. We do not include a fitting method yet so that the fitting is simply done by eye. We find that the slow and median rotator models have almost the same parametrization which is significantly different from that of the fast rotator model. This is particularly the case for the values of the  $K_1$  parameter used for the three rotator models which is somehow related to the braking efficiency :  $dJ/dt \propto K_1^2$ . Given the Table 1, it seems that the slow/median rotators need a more efficient braking ( $K_1 = 1.7$ ) compare to the fast rotators ( $K_1 = 1.55$ ). This suggests that a change appears between the slow/median rotator models and the fast ones which may be due to a change of the magnetic field topology.

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